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Double Compound Combination Anti-synchronization In A Non Identical Fractional Order Hyper Chaotic System

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Abstract
This paper proposes a new synchronization scheme called Double Compound Combination Anti-Synchronization(DCCAS). Here we have taken eight non identical fractional order hyper chaotic systems from which we take four master systems and other four slave systems.This technique is based on double compound synchronization and compound combination synchronization.Theses systems are synchronized using Lyapunov Stability Theory and Active control method.Due to complexity of the dynamical systems involved, this scheme would provide high security in transmitting and receiving signals.

Keywords
Double compound synchronization;Compound Combination synchronization;fractional order chaotic system;active control;Lyapunov Stablity Theory

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Contents

1. Introduction

The chaotic behavior as found in various natural and non-natural systems is a rich non-linear phenomenon and plays an important role across various disciplines. A hyper chaotic attractor is typically defined as having chaotic behavior with atleast two positive lyapunov exponents.The minimal dimension for continuous hyper chaotic system is four. The first 4-D flow hyper chaotic was proposed in 1979 by Rossler. Compound combination synchronization of chaos has been investigated using six chaotic system evolving from different initial conditions [1] .Whereas double compound synchronization between four drive system and two response system for memristor based Lorenz System have been investigated in [2].Since synchronization about combination of two master system and combination of two response systems have been studied in [3]. Complete synchronization of chaotic attractors of two identical complex Lorenz systems have been discussed in [4].The synchronization between two fractional order hyper chaotic systems have been studied in [5].Synchronization of non identical fractional order hyper chaotic systems are discussed in [6].A new fractional order hyper chaotic Rabinovich system and its dynamical behavior are discussed in [7].A novel fractional order hyper chaotic system with a quadratic exponential term and its synchronization are referred in [8].Chaotic analysis and combination combination synchronization of a novel hyper chaotic system without any equilibrium are in [9].To study on fractional order hyper chaotic complex system and their generalized function projective synchronization,a scheme based on the tracking control technique are in [10]. Generalization of combination combination synchronization of n-dimensional time delay chaotic system via robust adaptive sliding mode control, hyper chaos control and adaptive synchronization with uncertain parameter for fractional order Vander pol systems are discussed in [11] and [12]. As the double compound combination synchronization for eight identical memristor based integer order chua oscilators is investigated by using Lyapunov Stablity theory in [13].Analysis of hyper chaotic complex Lorenz system and the fractional lyapunov dimension of the hyper chaotic attractors of these system is calculate in [14] and the hybrid chaos synchronization of identical Arenedo system and non identical Arenedo and Rossler system in [15].The qualitative properties of the novel hyper chaotic Rikitake dynamo system are discussed in [16].Dual combination synchronization schemes for non identical different dimensional fractional order systems using scaling matrix has been investigated in [17].Synchronization and anti-synchronization of a fractional delayed memristor based chaotic system has been analyzed carefully.Whereas synchronization between non autonomous Liu and 4-D hyper chaotic system in the presence of uncertain parameter through active control method. In this manuscript our aim is to study the double compound combination anti synchronization(DCCAS) of eight non identical fractional order hyper chaotic systems.The paper has been organized as follows.Section 2 contain problem formulation,in which we have introduced the scheme of DCCAS.Section 3 contains system description and section 4 contains simulation results and section 5 concludes the article.

[1].

2. Problem Formulation

Double Compound Combination Anti-Synchronization Scheme
We now introduce the scheme of double compound combination anti synchronization (DCCAS) among four n dimensonal master systems and four n-dimensional slave systems .
We consider the first two master systems as:

$$D^{\alpha}x_1 = f_1(x_1(t)) \quad (1)$$
$$D^{\alpha}x_2 = f_2(x_2(t)) \quad (2)$$

Next we take two base master systems as:

$$D^{\alpha}x_3 = f_3(x_3(t)) \quad (3)$$
$$D^{\alpha}x_4 = f_4(x_4(t)) \quad (4)$$

Corresponding to the first two master systems we take two slave systems as:

$$D^{\alpha}y_1 = g_1(y_1(t)) + u_1 \quad (5)$$
$$D^{\alpha}y_2 = g_2(y_2(t)) + u_2 \quad (6)$$

Next two slave systems are taken as:

$$D^{\alpha}y_3 = g_3(y_3(t)) + u_3 \quad (7)$$
$$D^{\alpha}y_4 = g_4(y_4(t)) + u_4 \quad (8)$$

Here $x_i = diag(x_{i1}, x_{i2}, ..., x_{in})$ and $y_i = diag(y_{i1}, y_{i2}, ..., y_{in})$ are the state variables of the system (1)-(8) respectively.

$f_i(x_i) = diag(f_{i1}(x_i), f_{i2}(x_i), ..., f_{in}(x_i))$, $g_i(y_i) = diag$ $(g_{i1}(y_i), g_{i2}(y_i), ..., g_{in}(y_i))$ for i=1,2,3,4 are continuous functions of master and slave systems (1)-(8).

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$u_i, i = 1,2,3,4$ are control functions to be designed.

Definition: Systems(1)-(8) are said to be in double compound combination anti-synchronization , if the error defined as $e = (A_1x_1 + A_2x_2)(A_3x_3 + A_4x_4) + B_1y_1 + B_2y_2 + B_3y_3 + B_4y_4$ tends to zero,i.e.

$$lim_{t\to\infty}\|e\| = lim_{t\to\infty}\|(A_1x_1 + A_2x_2)(A_3x_3 + A_4x_4) + B_1y_1 + B_2y_2 + B_3y_3 + B_4y_4\| = 0 \quad (9)$$

where $A_i = diag(a_{i1}, a_{i2}, ..., a_{in}), B_i = diag(b_{i1}, b_{i2}, ..., b_{in})$,i=1,2,3,4,$\|.\|$ is the matrix norm and $e = diag(e_1, e_2, ..., e_n)$
In particular,if we take $A_i = diag(1,1,...,1), B_i = diag(1,1,....1)$
Then the error can be written as

$$e = (x_1 + x_2)(x_3 + x_4) + (y_1 + y_2 + y_3 + y_4) \quad (10)$$

Therefore we get the error system as:

$$D^{\alpha}e = (D^{\alpha}x_1 + D^{\alpha}x_2)(x_3 + x_4) + (x_1 + x_2)(D^{\alpha}x_3 + D^{\alpha}x_4) + (D^{\alpha}y_1 + D^{\alpha}y_2 + D^{\alpha}y_3 + D^{\alpha}y_4) \quad (11)$$

Putting the values of the derivatives we get:

$$D^{\alpha}e = ([(h_{1i} + h_{21})(x_{3i} + x_{4i}) + (x_{1i} + x_{2i})(h_{3i} + h_{4i}) + j_{1i} + j_{2i} + j_{3i} + j_{4i} + u_i] \quad (12)$$

Theorem 1 If the Control functions are chosen of the form

$$u_i = -(h_{1i} + h_{2i})(x_{3i} + x_{4i}) - (x_{1i} + x_{2i})(h_{3i} + h_{4i}) - g_{1i} - g_{2i} - g_{3i} - g_{4i} - Ke$$

Then the master systems and slave systems achieve DCCAS.
Here K=diag($K_1, K_2, ..., K_n$)is a positive definite matrix.
Proof: We define the Lyapunov function as:

$$V(e) = \frac{1}{2}\sum e^2$$
$$D^{\alpha}V(e) = \sum eD^{\alpha}e$$
$$= \sum e[(h_{1i} + h_{21})(x_{3i} + x_{4i}) + (x_{1i} + x_{2i})(h_{3i} + h_{4i}) + j_{1i} + j_{2i} + j_{3i} + j_{4i} + u_i$$

Substituting the designed controllers u_i from Theorem 1 in (12),we get $D^{\alpha}V(e) = -\sum Ke^2$
i.e.$D^{\alpha}V$ is negative definite.
Therefore by Lyapunov Stablity Theorem , $lim_{t\to\infty}\|e\| =$

0 This establishes that the master systems (1-4) and slave systems(5-8) are now anti-synchronized in double compound combination manner.

Remarks:

1.If two of $B_i,(i=1,2,3,4)=0$, then DCCAS reduces to double compound anti-synchronization.

2. If one of $A_i,(i=1,2,3,4)=0$ and two of $B_i,(i=1,2,3,4)=0$ then it reduces to compound combination anti-synchronization.

3.For the choice $A_i=0$ and three of $B_i, i=1,2,3,4=0$,then chaos control can be obtained.

4.The synchronization of (i) one drive and one response chaotic systems (ii)two drive and one response systems (iii)two drive and two response systems and so on are special case of our eight chaotic systems.

5. The error $e=(A_1x_1+A_2x_2)(A_3x_3+A_4x_4)+B_1y_1+B_2y_2+B_3y_3+B_4y_4$ can be written as: $e=[A_1x_1(A_3x_3+A_4x_4)+B_1y_1+B_2y_2]+[A_2x_2(A_3x_3+A_4x_4)+B_3y_3+B_4y_4]$ which can be considered as the sum of two error of the anti synchronization of compound combination.

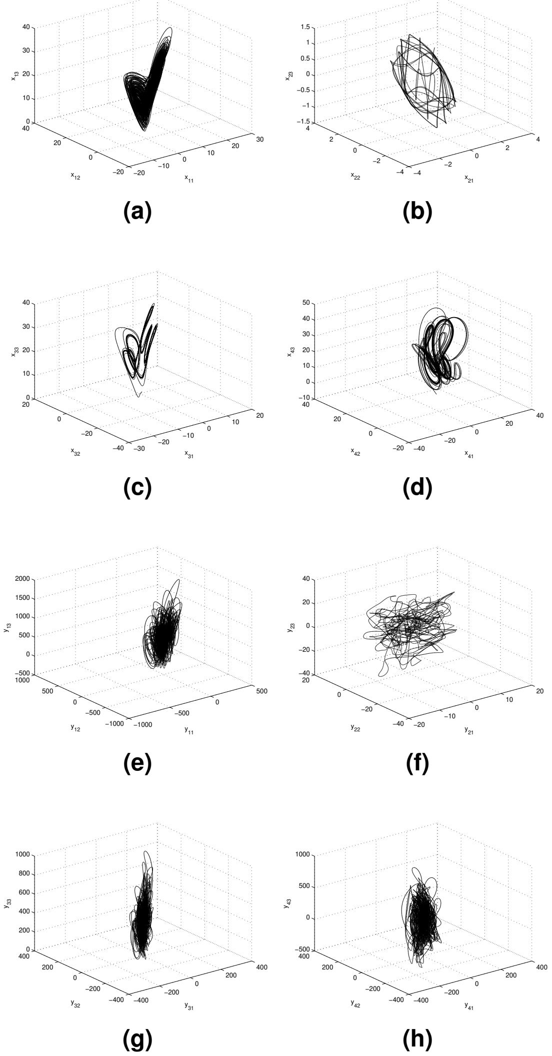


Fig 1::Phase Portraits of fractional order hyper chaotic (a)Lu system in $x_{11}-x_{12}-x_{13}$ (b)4D Integral system in $x_{21}-x_{22}-x_{23}$ (c)Chen system in $x_{31}-x_{32}-x_{33}$ (d)Lorenz system in $x_{41}-x_{42}-x_{43}$ (e)Xling system in $y_{11}-y_{12}-y_{13}$ (f)Vanderpol system in $y_{21}-y_{22}-y_{23}$ (g)Rabinovich system in $y_{31}-y_{32}-y_{33}$ (h)Rikitake system in $y_{41}-y_{42}-y_{43}$

3. System Description

Fractional Order Hyper Chaotic Lu System:

$$\begin{aligned}
\frac{d^\alpha x_{11}}{dt^\alpha} &= a_{11}(x_{12}-x_{11})+x_{14}\\
\frac{d^\alpha x_{12}}{dt^\alpha} &= -x_{11}x_{13}+a_{13}x_{12}\\
\frac{d^\alpha x_{13}}{dt^\alpha} &= x_{11}x_{12}-a_{12}x_{13}\\
\frac{d^\alpha x_{14}}{dt^\alpha} &= x_{11}x_{13}+a_{14}x_{14}
\end{aligned} \tag{13}$$

For $a_{11}=36, a_{12}=3, a_{13}=20, a_{14}=-1$ and $\alpha=0.95$ this system is hyper chaotic for initial conditions $(x_{11},x_{12},x_{13},x_{14})$=(-10,-14,12,10) as displayed in Fig. 1.

Fractional Order 4D Integral Hyper Chaotic System:

$$\begin{aligned}
\frac{d^\alpha x_{21}}{dt^\alpha} &= a_{21}x_{21}-x_{22}\\
\frac{d^\alpha x_{22}}{dt^\alpha} &= x_{21}-x_{22}x_{23}x_{23}\\
\frac{d^\alpha x_{23}}{dt^\alpha} &= -a_{22}x_{22}-a_{21}x_{21}-a_{23}x_{23}-a_{24}x_{24}\\
\frac{d^\alpha x_{24}}{dt^\alpha} &= x_{23}+a_{25}x_{24}
\end{aligned} \tag{14}$$

For $a_{21}=0.56, a_{22}=1.0, a_{23}=1.0, a_{24}=6, a_{25}=0.8$ and $\alpha=0.95$, this system is hyper chaotic for initial conditions

$(x_{21},x_{22},x_{23},x_{24})$=(1.2,0.6,0.8,0.5) as displayed in Fig. 1.

Fractional Order Hyper Chaotic Chen System:

$$\begin{aligned}\frac{d^\alpha x_{31}}{dt^\alpha} &= a_{31}(x_{32}-x_{31})+x_{34}\\ \frac{d^\alpha x_{32}}{dt^\alpha} &= a_{32}x_{31}-x_{31}x_{33}+a_{33}x_{32}\\ \frac{d^\alpha x_{33}}{dt^\alpha} &= x_{31}x_{32}-a_{34}x_{33}\\ \frac{d^\alpha x_{34}}{dt^\alpha} &= x_{32}x_{33}+a_{35}x_{34}\end{aligned} \tag{15}$$

For $a_{31}=35, a_{32}=7, a_{33}=12, a_{34}=3, a_{35}=0.3$ and $\alpha=0.95$, this system is hyper chaotic for initial conditions $(x_{31},x_{32},x_{33},x_{34})$=(-1,-3,2,5)as displayed in Fig. 1.

Fractional Order Hyper Chaotic Lorenz System:

$$\begin{aligned}\frac{d^\alpha x_{41}}{dt^\alpha} &= a_{41}(x_{42}-x_{41})+x_{44}\\ \frac{d^\alpha x_{42}}{dt^\alpha} &= a_{42}x_{41}-x_{42}-x_{41}x_{43}\\ \frac{d^\alpha x_{43}}{dt^\alpha} &= x_{41}x_{42}-a_{43}x_{43}\\ \frac{d^\alpha x_{44}}{dt^\alpha} &= -x_{42}x_{43}+a_{44}x_{44}\end{aligned} \tag{16}$$

For $a_{41}=10, a_{42}=28, a_{43}=8/3, a_{44}=-1$ and $\alpha=0.95$,this system is hyper chaotic for initial conditions $(x_{41},x_{42},x_{43},x_{44})$= (1.5,3,-1,3) as displayed in Fig. 1.

Fractional Order Hyper Chaotic System (proposed by Xin and Ling) [19]

$$\begin{aligned}\frac{d^\alpha y_{11}}{dt^\alpha} &= b_{11}(y_{12}-y_{11})+y_{14}\\ \frac{d^\alpha y_{12}}{dt^\alpha} &= b_{12}y_{11}+y_{11}y_{13}-y_{14}\\ \frac{d^\alpha y_{13}}{dt^\alpha} &= -b_{13}y_{13}-b_{14}y_{11}y_{11}\\ \frac{d^\alpha y_{14}}{dt^\alpha} &= b_{13}y_{11}\end{aligned} \tag{17}$$

For $b_{11}=10, b_{12}=40, b_{13}=2.5, b_{14}=4$ and $\alpha=0.95$,this system is hyper chaotic for initial conditions $(y_{11},y_{12},y_{13},y_{14})$ =(1,2,3,4)as displayed in Fig. 1.

Fractional Order Vanderpol Hyper Chaotic System: [12]

$$\begin{aligned}\frac{d^\alpha y_{21}}{dt^\alpha} &= y_{22}\\ \frac{d^\alpha y_{22}}{dt^\alpha} &= -(b_{21}+b_{22}y_{23})y_{21}-(b_{21}+b_{22}y_{23})y_{21}^3-b_{23}y_{22}\end{aligned} \tag{18}$$

$$\begin{aligned}&+b_{24}y_{23}\\ \frac{d^\alpha y_{23}}{dt^\alpha} &= y_{24}\\ \frac{d^\alpha y_{24}}{dt^\alpha} &= -b_{25}y_{23}+b_{26}(1-y_{23}y_{23})y_{24}+b_{27}y_{21}\end{aligned} \tag{19}$$

For $b_{21}=10, b_{22}=3, b_{23}=0.4, b_{24}=70, b_{25}=1, b_{26}=5, b_{27}=0.1$and $\alpha=0.95$,this system is hyper chaotic for initial conditions $(y_{21},y_{22},y_{23},y_{24})$=(0.1,-0.5,0.1,-0.5) as displayed in Fig. 1.

Fractional Order Rabinovich Hyper Chaotic System: [7]

$$\begin{aligned}\frac{d^\alpha y_{31}}{dt^\alpha} &= -b_{31}y_{31}+b_{32}y_{32})+y_{32}y_{33}\\ \frac{d^\alpha y_{32}}{dt^\alpha} &= b_{32}y_{31}-b_{33}y_{32}-y_{31}y_{33}+y_{34}\\ \frac{d^\alpha y_{33}}{dt^\alpha} &= -b_{34}y_{33}+y_{31}y_{32}\\ \frac{d^\alpha y_{34}}{dt^\alpha} &= -b_{35}y_{32}\end{aligned} \tag{20}$$

For $b_{31}=34, b_{32}=6.75, b_{33}=1, b_{34}=1, b_{35}=2$ and $\alpha=0.95$,this system is hyper chaotic for initial conditions $(y_{31},y_{32},y_{33},y_{34})$= (5.5,-1.25,8.4,2.75) as displayed in Fig. 1.

Fractional Order Hyper Chaotic Rikitake Dynamic System: [16]

$$\begin{aligned}\frac{d^\alpha y_{41}}{dt^\alpha} &= -b_{41}y_{41}+y_{42}y_{43}-b_{42}y_{44}\\ \frac{d^\alpha y_{42}}{dt^\alpha} &= -b_{41}y_{42}+y_{41}(y_{43}-b_{43})-b_{42}y_{44}\\ \frac{d^\alpha y_{43}}{dt^\alpha} &= 1-y_{41}y_{42}\\ \frac{d^\alpha y_{44}}{dt^\alpha} &= b_{44}y_{42}\end{aligned} \tag{21}$$

For $b_{41}=1, b_{42}=1.7, b_{43}=1, b_{44}=0.7$ and $\alpha=0.95$, this system is hyper chaotic for initial conditions $(y_{41},y_{42},y_{43},y_{44})$=(3.5,1.7,-4.5,2.8) as displayed in Fig. 1.

4. Numerical Simulations & Discussions

From (10), we have the error as:

$$e=(A_1x_1+A_2x_2)(A_3x_3+A_4x_4)+(B_1y_1+B_2y_2+B_3y_3+B_4y_4) \tag{22}$$

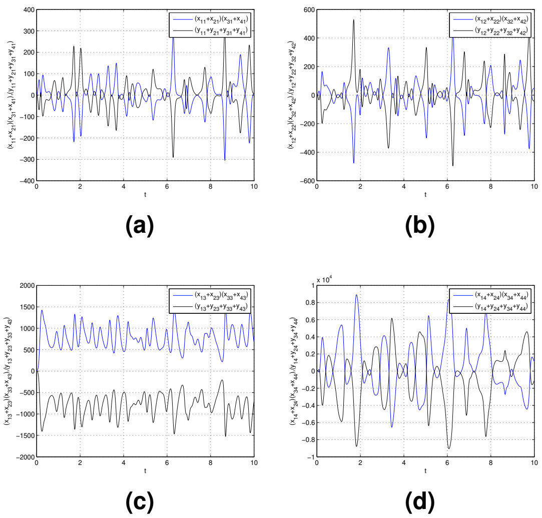


Fig2: Double compound combination anti-synchronized trajectories of master and slave systems

Therefore, the error system is:

$$\begin{aligned}
e_{11} &= (x_{11}+x_{21})(x_{31}+x_{41})+y_{11}+y_{21}+y_{31}+y_{41}\\
e_{12} &= (x_{12}+x_{22})(x_{32}+x_{42})+y_{12}+y_{22}+y_{32}+y_{42}\\
e_{13} &= (x_{13}+x_{23})(x_{33}+x_{43})+y_{13}+y_{23}+y_{33}+y_{43}\\
e_{14} &= (x_{14}+x_{24})(x_{34}+x_{44})+y_{14}+y_{24}+y_{34}+y_{44}
\end{aligned}$$

Differentiating the above we get the following:

$$\begin{aligned}
D^{\alpha}e_{11} = &\ (D^{\alpha}x_{11}+D^{\alpha}x_{21})(x_{31}+x_{41})+(x_{11}+x_{21})\\
&(D^{\alpha}x_{31}+D^{\alpha}x_{41})+(D^{\alpha}y_{11}+D^{\alpha}y_{21}+D^{\alpha}y_{31}\\
&+D^{\alpha}y_{41})\\
D^{\alpha}e_{12} = &\ (D^{\alpha}x_{12}+D^{\alpha}x_{22})(x_{32}+x_{42})+(x_{12}+x_{22})\\
&(D^{\alpha}x_{32}+D^{\alpha}x_{42})+(D^{\alpha}y_{12}+D^{\alpha}y_{22}+D^{\alpha}y_{32}\\
&+D^{\alpha}y_{42})\\
D^{\alpha}e_{13} = &\ (D^{\alpha}x_{13}+D^{\alpha}x_{23})(x_{33}+x_{43})+(x_{13}+x_{23})\\
&(D^{\alpha}x_{33}+D^{\alpha}x_{43})+(D^{\alpha}y_{13}+D^{\alpha}y_{23}+D^{\alpha}y_{33}\\
&+D^{\alpha}y_{43})\\
D^{\alpha}e_{14} = &\ (D^{\alpha}x_{14}+D^{\alpha}x_{24})(x_{34}+x_{44})+(x_{14}+x_{24})\\
&(D^{\alpha}x_{34}+D^{\alpha}x_{44})+(D^{\alpha}y_{14}+D^{\alpha}y_{24}+D^{\alpha}y_{34}\\
&+D^{\alpha}y_{44}) \qquad (23)
\end{aligned}$$

Now designing the controllers as in Theorem 1:

$$\begin{aligned}
u_1 = &-(a_{11}(x_{12}-x_{11})+x_{14})(x_{31}+x_{41})-(b_{11}(y_{12}-y_{11})\\
&+y_{14})-K_1(x_{11}x_{41}+y_{11})-(a_{21}x_{21}-x_{22})(x_{31}\\
&+x_{41})-y_{22}-K_1(x_{21}x_{31}+y_{21})-(a_{31}(x_{32}-x_{31})\\
&+x_{34})(x_{31}+x_{41})-(-b_{31}y_{31}+b_{32}y_{32}+y_{32}y_{33})\\
&+y_{14})-K_1(x_{11}x_{21}+y_{31})-(a_{41}(x_{42}-x_{41})+x_{44})\\
&(x_{11}+x_{21})-y_{22}-K_1(x_{21}x_{41}+y_{41})
\end{aligned}$$

$$\begin{aligned}
u_2 = &-(-x_{11}x_{13}+a_{13}x_{12})(x_{32}+x_{42})-(b_{12}y_{11}+y_{11}\\
&y_{13}-y_{14})+y_{14})-K_2(x_{12}x_{42}+y_{12})-(x_{21}-x_{22}\\
&x_{23}x_{23})(x_{32}+x_{42})-[-(b_{21}+b_{22}y_{23})y_{21}-(b_{21}\\
&+b_{22}y_{23})y_{21}y_{21}y_{21}-b_{23}y_{22}+b_{24}y_{23}]\\
&-K_2(x_{22}x_{32}+y_{22})-(a_{32}x_{31}-x_{31}x_{33}+a_{33}\\
&x_{32})(x_{12}+x_{22})-(b_{32}y_{31}-b_{33}y_{32}-y_{31}y_{33})\\
&-K_2(x_{12}x_{32}+y_{32})-(a_{42}x_{41}-x_{42}-x_{41}x_{43})\\
&(x_{12}+x_{22})-(-b_{41}y_{42}+y_{41}(y_{43}-b_{43})-b_{42}y_{44})\\
&-K_2(x_{22}x_{42}+y_{42} \qquad (24)
\end{aligned}$$

$$\begin{aligned}
u_3 = &-(x_{11}x_{12})(x_{33}+x_{43})-(-b_{13}y_{13}-b_{14}y_{11}y_{11})\\
&-K_3(x_{13}x_{43}+y_{13})-(-a_{22}x_{22}-a_{21}x_{21}-a_{23}\\
&x_{23}-a_{24}x_{24})(x_{33}+x_{43})-[y_{24}]-K_3(x_{23}x_{33}\\
&+y_{23})-(x_{31}x_{32}-a_{34}x_{33})(x_{13}+x_{23})-(-b_{34}y_{33}\\
&+y_{31}y_{32})-K_3(x_{13}x_{33}+y_{33})-(x_{41}x_{42}-a_{43}\\
&x_{43})(x_{13}+x_{23})-(1-y_{41}y_{42}+y_{41})-K_3(x_{23}x_{43}+y_{43})
\end{aligned}$$

$$\begin{aligned}
u_4 = &-(x_{11}x_{13}+a_{14}x_{14})(x_{34}+x_{44})-(b_{13}y_{11})-K_4\\
&(x_{14}x_{44}+y_{14})-(x_{23}+a_{25}x_{24})(x_{34}+x_{44})\\
&-[-b_{25}y_{23}+b_{26}(1-y_{23}y_{23})y_{24}+b_{27}y_{21}]\\
&-K_4(x_{24}x_{34}+y_{24})-(x_{32}x_{33}+a_{35}x_{34})(x_{14}+x_{24})\\
&-(-b_{35}y_{32})-K_4(x_{14}x_{34}+y_{34})-(-x_{42}x_{43}+a_{44}\\
&x_{44})(x_{14}+x_{24})-(b_{44}y_{42})-K_4(x_{24}x_{44}+y_{44})
\end{aligned}$$

Applying the controllers in equation (25),the error dynamics simplifies as:

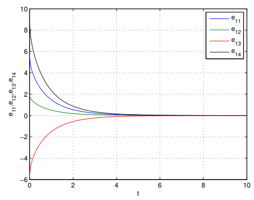
$$\begin{aligned}
D^{\alpha}e_{11} &= -K_1e_{11}\\
D^{\alpha}e_{12} &= -K_2e_{12}\\
D^{\alpha}e_{13} &= -K_3e_{13}\\
D^{\alpha}e_{14} &= -K_4e_{14}
\end{aligned} \qquad (25)$$

Next we consider the Lyapunov function as:

$$V=\frac{1}{2}\sum_{i=1}^{4}e_i^2$$

Clearly V is positive definite function with a negative definite derivative.

$$\begin{aligned}
D^{\alpha}V &= \sum_{i=1}^{4}e_i(D^{\alpha}e_i)\\
&= \sum_{i=1}^{4}e_i(-K_ie_i)
\end{aligned}$$



(a)

Fig 3: Double compound combination anti-synchronization errors

$$-\sum_{i=1}^{4} K_i e_i^2 < 0$$

Therefore by Lyapunov Stability Theorem we have that error asymptotically converges to 0,i.e. double compound combination anti synchronization is achieved.

For numerical Simulations we have considered $K_i = 1 \forall i = 1,2,3,4$. The double compound combination anti-synchronized trajectories have been displayed in Fig. 2.The errors converging to zero has been shown in Fig.3.

5. Conclusion

In this paper DCCAS has been achieved among eight chaotic systems. We have used the Lyapunov Stablity Theory and Active Control method to achieve DCCAS.By designing suitable controllers the error converges to zero.Such technique would prove fruitful in the field of secure communication because of the complexity of the systems involved.

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